

# Optimal Defensive Allocations in the Face of Uncertain Terrorist Preferences, with an Emphasis on Transportation

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## ABSTRACT

*This paper extends a game-theoretic model for identifying optimal defensive resource allocations to the case of realistic multi-attribute terrorist objective functions. In particular, we compare the optimal defensive resource allocations to ten major US urban areas in the face of uncertain terrorist preferences with and without transportation-related attributes. The defender's uncertainty about terrorist preferences is addressed both by probability distributions over the attacker's attribute weights, and by allowing for attributes that are important to the attacker but not known to the defender. Estimates of the various terrorist attribute weights are inferred from (partial) ordinal expert judgments using the technique of probabilistic inversion.*

## INTRODUCTION

Allocating a limited budget to protect potential targets against terrorist attackers is an important but difficult task. In doing so, we must take into account both the strategic nature of the attackers, and also the defender's uncertain knowledge about attacker preferences. A variety of game-theoretic models in the face of defender uncertainty have been studied and applied.<sup>1</sup>

This study is based on the sequential game with incomplete information developed by Bier et al.<sup>2</sup> In that model, the defender moves first to allocate her defensive resources among the potential targets under uncertain knowledge about attacker preferences; the attacker then selects the target with the highest payoff to attack, in light of any defensive investments.<sup>3</sup> Knowing or assuming that the attacker would play his best response to any given defensive allocation, the defender wishes to choose her allocation so as to effectively protect against attacks, deter attacks, or deflect attacks to

less important targets. However, with uncertainty about the attacker's utility function, the defender cannot predict the attacker's best response for sure; therefore, the defender is assumed to minimize her expected total loss (where the defender objectives may in general be different from the attacker objectives).

This paper extends the above game-theoretic model for determining optimal defensive resource allocations to the case of more realistic multi-attribute terrorist objective functions. In particular, we compare the optimal defensive resource allocations in the face of uncertain terrorist preferences with and without transportation-related attributes. The defender's uncertainty about terrorist preferences is represented both by probability distributions over the attacker's attribute weights, and by allowing for attributes that are important to the attacker but not known to the defender.

One closely related task is to elicit the attacker attribute weights in the terrorist multi-attribute objective from the judgments of intelligence experts. However, direct estimation of attribute weights can be difficult, since intelligence analysts are usually not familiar with utility theory, and historical data about terrorist attacks are relatively sparse. In such cases, indirect elicitation may be preferable. In particular, this paper uses an approach in which experts are asked to give (partial) rank orderings of attack strategies or targets, and the attribute weights in the attacker objective function are then inferred from those partial rankings using probabilistic inversion.<sup>4</sup> We believe that this approach will increase the acceptance of quantitative methods by intelligence experts, and also make it possible to elicit the opinions of a large number of experts in an automated (e.g., online) manner.

**MODEL**

As in Wang and Bier,<sup>5</sup> we assume that the defender’s objective is to minimize the total expected loss, as given by

$$\min_{c_1, \dots, c_n} \sum_{i=1}^n h_i(c_1, \dots, c_n) p(c_i) v_i \quad \text{such that} \quad \sum_{i=1}^n c_i \leq B$$

where:

- $n$  = number of targets
- $c_i$  = defender’s resource allocation to target  $i$
- $B$  = defender’s total budget
- $v_i$  = defender’s valuation of target  $i$
- $h_i(c_1, \dots, c_n)$  = probability of an attack on target  $i$
- $p(c_i) = e^{-\lambda c_i}$  = success probability of an attack on target  $i$ , as a function of the budget allocated to target  $i$ , where  $\lambda$  is the cost effectiveness of defensive investment.<sup>6</sup>

The attacker is then assumed to observe the defender’s resource allocations  $c_i$  and then choose the target with the highest payoff in light of any defensive investment:

$$\max_i p(c_i) U_i$$

where:

- $U_i = \sum_{j=1}^{m-1} x_j u_j(A_{ij}) x_j + e_i x_m =$  attacker’s utility of target  $i$

- $x_j$  = attacker weight on attribute  $j$  ( $x_j \geq 0, j=1, \dots, m$ , and  $\sum_{j=1}^m x_j = 1$ )
- $A_{ij}$  = attacker rating of target  $i$  on attribute  $j$  ( $j=1, \dots, m-1$ )
- $u_j$  = single-attribute utility function for attribute  $j$ , taking on values in  $[0, 1]$ .
- $\varepsilon_i$  = attacker utility of target  $i$  on the unobserved attribute (modeled as independent, identically uniformly distributed random variables taking on values in  $[0, 1]$ ).

Experts are asked to give partial rank orderings of the various possible attacker targets or strategies (e.g., the top five and bottom five). The probability distributions of the various attribute weights are then estimated using probabilistic inversion.<sup>7</sup>

**CASE STUDY**

We conduct a case study on the ten major US urban areas with the highest expected damage from terrorism,<sup>8</sup> based on two different sets of attributes: “macro” attributes (expected property losses from terrorism, and total population);<sup>9</sup> and transportation-related attributes (yearly air departures, and average daily bridge traffic on the most heavily traveled bridge).<sup>10</sup> The ten urban areas are: New York City (NYC); Chicago; San Francisco; Washington, DC; Los Angeles (LA); Philadelphia; Boston; Houston; Newark; and Seattle. The attribute values for these ten urban areas are presented in Table 1.

Urban Area	Expected Property Loss from Terrorism (\$ million)	Population	Yearly Air Departures	Average Daily Bridge Traffic
NYC	413	9,314,235	23,599	596,400
Chicago	115	8,272,768	39,949	318,800
San Francisco	57	1,731,183	19,142	277,700
Washington, DC	36	4,923,153	17,253	254,975
LA	34	9,519,338	28,816	336,000
Philadelphia	21	5,100,931	13,640	192,204
Boston	18	3,406,829	11,625	669,000
Houston	11	4,177,646	20,979	308,060
Newark	7.3	2,032,989	12,827	518,100
Seattle	6.7	2,414,616	13,578	212,000

**Table 1.** Attribute values for the ten urban areas with the highest expected terrorism losses.

We consider two hypothetical experts who give partial rank orderings of these urban areas. In particular, Expert 1 ranks NYC, Chicago, LA, San Francisco, and DC as the top five most attractive cities (in that order), and Houston at the bottom; Expert 2 ranks Chicago, LA, NYC, Houston, and Boston as the top five cities (in that order), and Philadelphia at the bottom.

For this case study, we assume that the attacker’s single-attribute utility function for attribute  $j$  ( $j = 1, \dots, m - 1$ ),  $u_j(A_{kj})$  is proportional to  $\ln(\frac{A_{kj}}{a_j})$  (where  $a_j = \min_i A_{ij}$ ), and hence measures how much more attractive target  $k$  is than the least desirable target on attribute  $j$ . They are normalized so that  $u_j(A_{kj}) = 1$  when target  $k$  is the most attractive target on attribute  $j$ . This choice of utility function is also consistent with Fechner’s law, which states that human

perceptions are typically logarithmic in the magnitude of the original stimuli.<sup>11</sup> For example, if the expected property loss in Chicago is doubled from 115 to 230 million dollars, the attacker’s single-attribute utility increases by an additive increment proportional to  $\ln(2)$ .

The histograms in Figure 1 show the probability distributions for the attacker attribute weights inferred from the rankings of expert 1, with and without transportation-related attributes. Including transportation-related attributes reduces the expected weight on the unobserved attribute from 31 percent to 24 percent. However, the weight on the unobserved attribute is still substantial, reflecting the fact that the rankings given by Expert 1 do not place a lot of importance on air departures and bridge traffic.

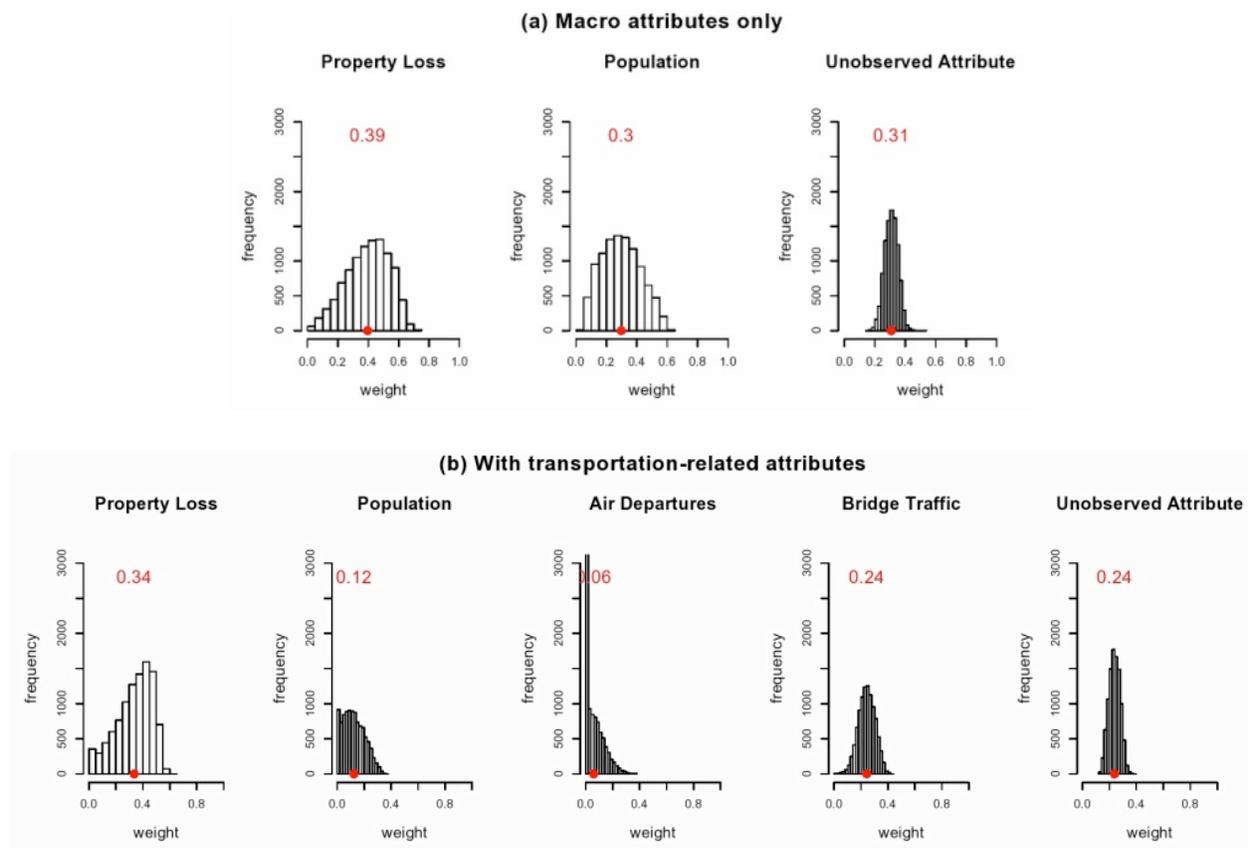
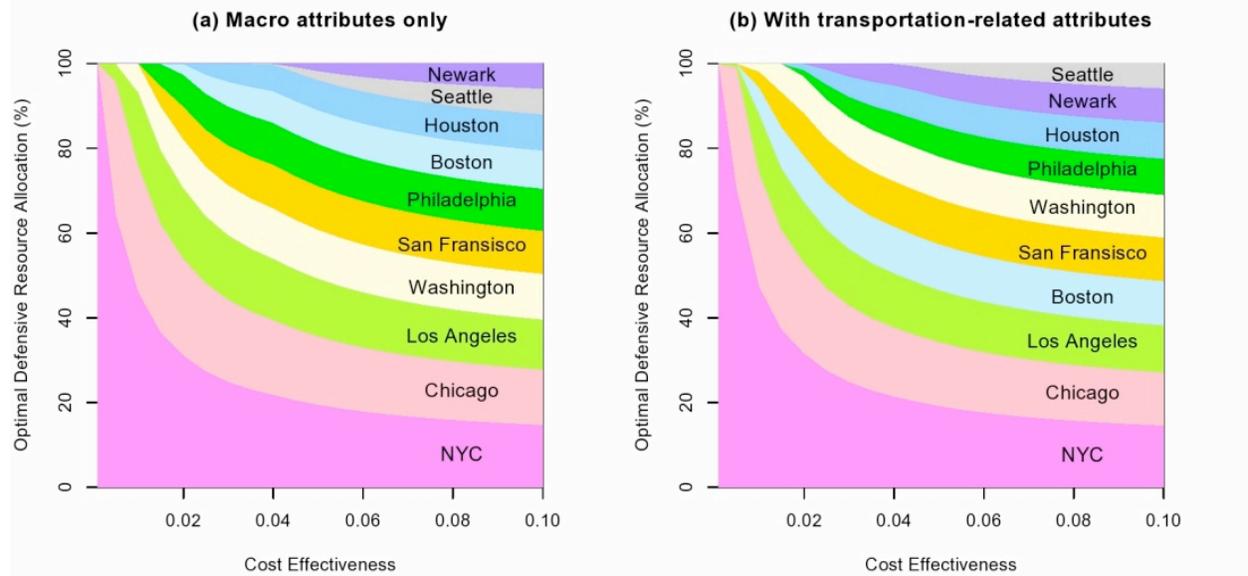


Figure 1. Attribute weights inferred from the rankings of Expert 1.

The optimal defensive resource allocations based on the elicited attribute weights from Expert 1 are shown in Figure 2. Note that San Francisco is rated higher than DC both by Expert 1, and on the defender objective function (property loss). However, as shown in Figure 2a, when using only the two macro attributes, DC gets more resources than San

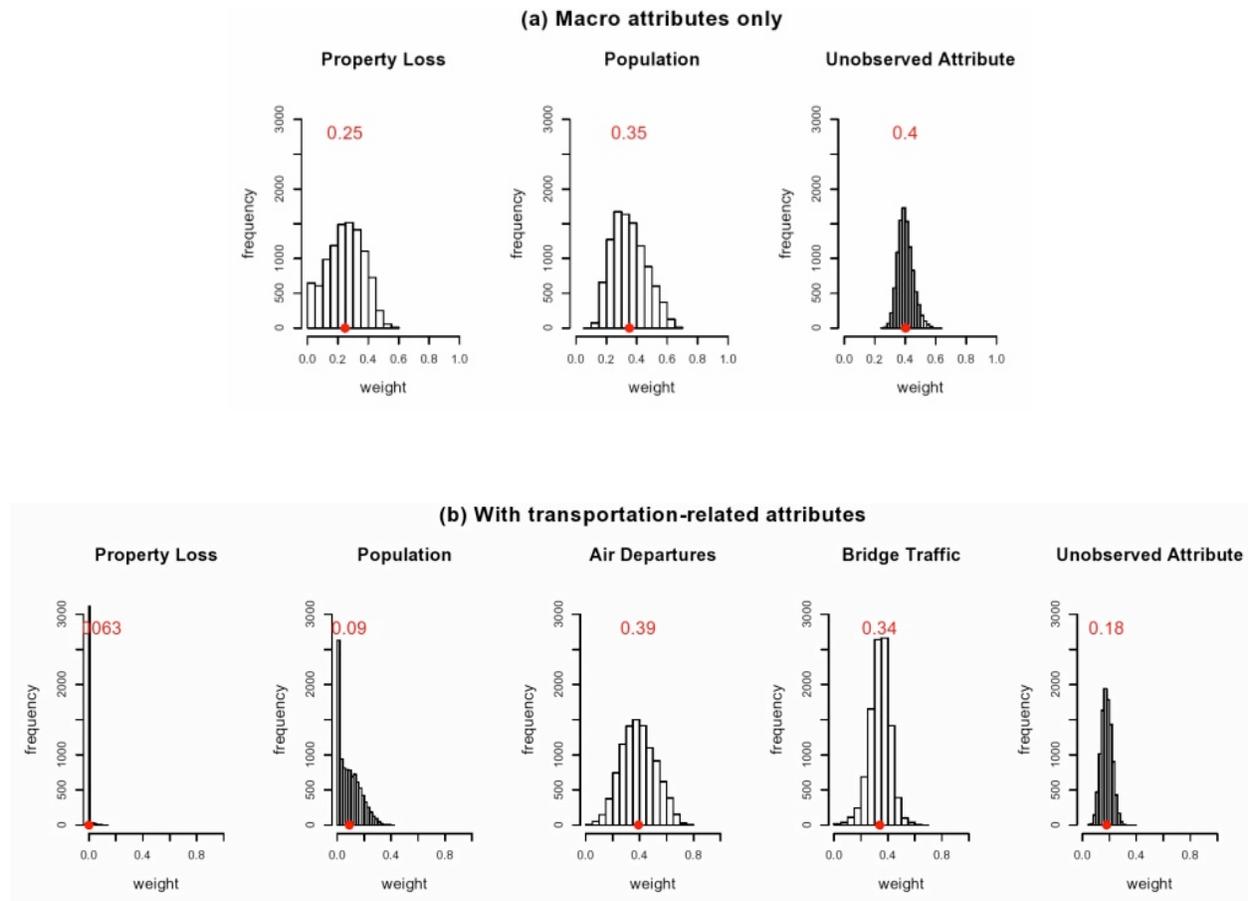
Francisco, reflecting the fact that the model does not have enough information to distinguish clearly between San Francisco and DC. Including the transportation-related attributes allows the model to perform better in this regard, as shown by the reversed ranking of San Francisco and DC in Figure 2b.



**Figure 2.** Optimal defensive allocations based on elicited attribute weights from Expert 1.

Figures 3 and 4 give comparable results for hypothetical Expert 2. Figure 3a shows that without transportation-related attributes, the model puts 40 percent of the weight on the unobserved attribute, indicating that the macro attributes are not sufficient to adequately represent the beliefs

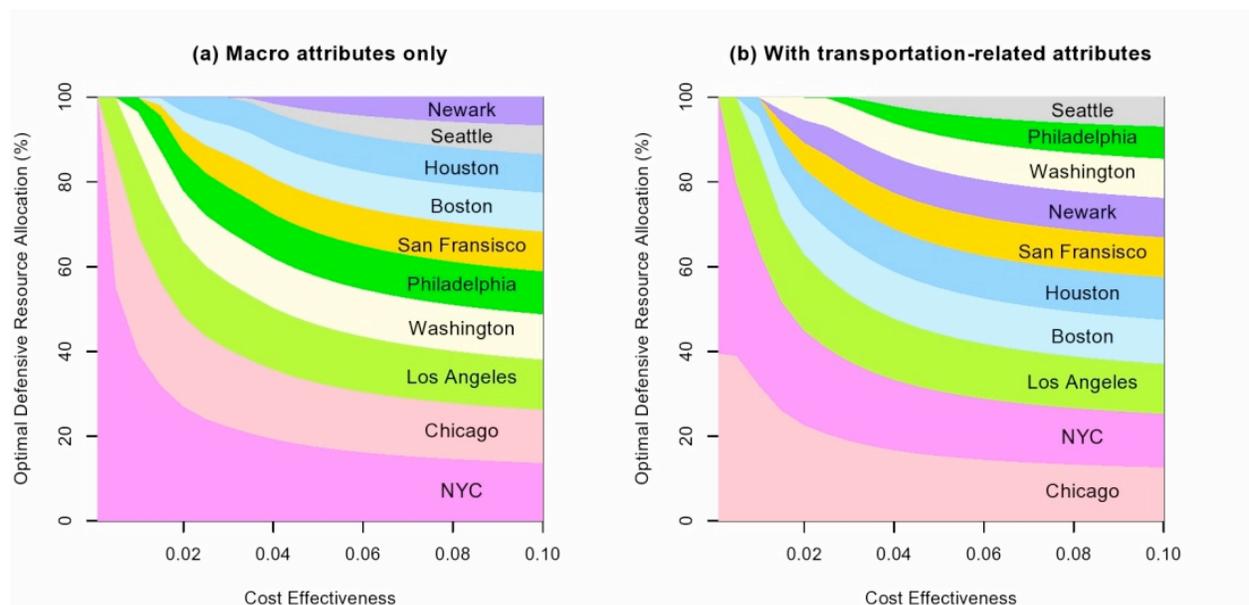
of Expert 2. By contrast, Figure 3b shows that the ratings given by Expert 2 are consistent with a high weight on transportation-related attributes. As a result, the weight on the unobserved attribute drops to only 18 percent.



**Figure 3.** Attribute weights inferred from the rankings of Expert 2.

Using only macro attributes also performs poorly for the optimal defensive allocations resulting from the judgments of Expert 2, since the expert’s target rankings (Chicago, LA, NYC, Houston, and Boston as the top five cities, and Philadelphia at the bottom) reflect high weights on transportation-related attributes. For example, Houston is ranked fourth in air departures, but receives only

modest funding in Figure 4a; by contrast, Philadelphia is ranked seventh in air departures and tenth in bridge traffic, but receives relatively high levels of funding in Figure 4a, because of its large population. With the inclusion of transportation-related attributes, the model does a much better job of matching the stated rankings given by Expert 2, as shown in Figure 4b.



**Figure 4.** Optimal defensive allocations based on elicited attribute weights from Expert 2

## CONCLUSION

Intelligence analysts are sometimes unable or unwilling to provide quantitative risk estimates.<sup>12</sup> This paper bridges this gap by providing a practical and methodologically credible way for risk analysts to obtain quantitative risk estimates from ordinal rankings provided by intelligence analysts. In particular, if experts find it difficult to estimate attacker attribute weights, indirect elicitation based on (partial) rank ordering of attack targets or strategies can help ease the elicitation burden. We believe that this approach will increase the acceptance of quantitative approaches by intelligence experts, and increase the number of experts whose opinions can be elicited in an automated (e.g., online) manner.

In addition, the inclusion of unobserved attributes makes it possible to use our model in a diagnostic manner, to indicate whether we have enough attributes. Finally, the results presented here (based on hypothetical expert judgments) show that including transportation-related attributes can help to distinguish between targets, especially when an expert believes that the attacker puts high weight on transportation-related attack strategies.

## ABOUT THE LEAD AUTHOR

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## ACKNOWLEDGMENT

The United States Department of Homeland Security supported this research through the National Center for Risk and Economic Analysis of Terrorism Events (CREATE) under award number 2010-ST-061-RE0001. However, any opinions, findings, and conclusions or recommendations in this document are those of the authors and do not necessarily reflect views of the US Department of Homeland Security, the University of Wisconsin-Madison, the University of Southern California, or CREATE.

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<sup>1</sup> See, for example T. Sandler and K. Siqueira, “Games and Terrorism: Recent Developments,” *Simulation and Gaming* 40, no. 2 (2009): 164-192; S.D. Guikema, “Game Theory Models of Intelligent Actors in Reliability Analysis: An Overview of the State of the Art,” in V. M. Bier, and N. Azaiez, eds. *Game Theoretic Risk Analysis of Security Threats* (New York: Springer, 2009).

<sup>2</sup> V.M. Bier, S. Oliveros, and L. Samuelson, “Choosing What to Protect: Strategic Defensive Allocation Against an Unknown Attacker,” *Journal of Public Economic Theory* 9, no. 4 (2007): 563-587.

<sup>3</sup> See also V.M. Bier, “Choosing What to Protect,” *Risk Analysis*, 27, no. 3 (2005): 607-620; J. Zhuang and V. M. Bier, “Balancing Terrorism and Natural Disasters – Defensive Strategy with Endogenous Attacker Effort,” *Operations Research* 55, no. 5 (2007): 976-991; V.M. Bier, V. M., N. Haphuriwat, J. Menoyo, R. Zimmerman, and A. M. Culpin, “Optimal Resource Allocation for Defense of Targets Based on Differing Measures of Attractiveness,” *Risk Analysis* 28, no. 3 (2008): 763-770, <http://www3.interscience.wiley.com/journal/119879163/abstract?CRETRY=1&SRETRY=0-fn1>; C. Wang and V. M. Bier, “Target-hardening Decisions Based on Uncertain Multi-attribute Terrorist Utility,” *Decision Analysis* 8, no. 4 (2011): 286-302.

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<sup>5</sup> C. Wang and V. M. Bier, “Target-hardening Decisions Based on Uncertain Multi-attribute Terrorist Utility,” *Decision Analysis* 8, no. 4 (2011): 286-302.

<sup>6</sup> Bier, et al., “Optimal Resource Allocation for Defense of Targets.”

<sup>7</sup> See note 4.

<sup>8</sup> According to H.H. Willis, A.R. Morral, T.K. Kelly, and J.J. Medby, *Estimating Terrorism Risk* (Santa Monica, CA: RAND Corporation, 2005, MG388).

<sup>9</sup> Ibid.

<sup>10</sup> Bier, et al., “Optimal Resource Allocation for Defense of Targets.”

<sup>11</sup> G.T. Fechner, *Elemente der Psychophysik (Elements of Psychophysics)* (Leipzig: Breitkopf und Härte, 1860).

<sup>12</sup> Homeland Security Studies and Analysis Institute, *Risk Analysis and Intelligence Communities Collaborative Framework* (2009), <http://www.homelanddefense.org/downloads/Risk-Intel%20Collaboration%20Final%20Report.pdf>.



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